SKEWNESS AND THE RELATION BETWEEN RISK AND RETURN

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Abstract

The relationship between risk and return has been one of the most important and extensively investigated issues in the financial economics literature. The theoretical results predict a positive relation between the two. Nevertheless, the empirical findings so far have been contradictory. Evidence presented in this paper show that these contradictions are the result of negative skewness in the distribution of portfolio excess return and the fact that the estimation of intertemporal asset pricing models are based on symmetric log-likelihood specifications.

Keywords: Risk-return tradeoff; SGT distribution; GARCH-M

JEL Classification: C18, C22, G15

1. Introduction

The financial and economic literature on the relationship between risk and return is voluminous and the findings thus far have been inconclusive. Many well known scholars have found a positive, others a negative and an equal number no relationship. For example, a significant positive risk-return relation for the US is reported in French et al. (1987), Lundblad (2007) and Lanne and Saikkonen (2007), a significant negative relation in Glosten et al. (1993), an insignificant one in Nelson (1991), Campbell and Hentschel (1992), Glosten et al. (1993), Theodossiou and Lee (1995) and Bansal and Lundblad (2002) and mixed findings in Baillie and DeGennaro (1990).[1]

In standard intertemporal capital asset pricing models, stochastic factors influence the investment opportunity set and through that the equilibrium risk premia of financial assets, e.g., Merton (1973). These factors trigger fluctuations in the risk-return tradeoff and as such, they are a source of skewness and kurtosis when returns are computed over discrete time intervals. Because investors hedge constantly against such fluctuations, higher moments are likely to be priced.

This paper investigates the impact of skewness and kurtosis on the risk-return relationship using an analytical framework based on the popular skewed generalized t (*SGT*) distribution, e.g., Theodossiou (1998). The *SGT* distribution is chosen because of its flexibility in modeling fat-tails, peakness and skewness, often observed in financial data.[2] Furthermore, it includes several well known symmetric distributions used in the finance literature, such as the generalized t (*GT*), generalized error (*GED*), student t (*T*) and normal (*N*), e.g., Bali and Theodossiou (2008) and Hansen et al. (2010).

2. Impact of Skewness on the Pricing of Risk

SGT Framework

The Intertemporal relationship between risk and returns is investigated using the GARCH-in-mean process, which has been the standard in the literature, e.g., Engle et al. (1987) and Glosten et al. (1993).

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[1] Contradictory findings are also reported in studies using other methodologies, such as Scruggs (1998), Harrison and Zhang (1999), Bali and Peng (2006), Ludvigson and Ng (2007), Pastór et al. (2008) and Chan et al. (1992).

[2] The SGT distribution has been used widely in finance for computing VaR measures, pricing options and estimating asset pricing models. It is also incorporated in econometric packages such as GAUSS.

That is, a portfolio's excess returns are specified as:

$$r_t = c\sigma_t + a + b\,r_{t-1} + u_t\,, \tag{1}$$

where σ_t^2 = var(r_t | I_{t-1}) is the conditional variance of r_t based on the information set I_{t-1} available prior to the realization of r_t, r_{t-1} is past value of excess returns included in I_{t-1}, a and b are typical regression coefficients and c, also known as the GARCH-in-mean coefficient, links σ_t to μ_t. For practical purposes and without loss of generality, a single lag value of r_t is used.

Under the SGT framework, r_t is modeled as

$$f\left(r_t|I_{t-1}\right) = \frac{k}{2}\left(\frac{n+1}{k}\right)^{-\frac{1}{k}} B\left(\frac{1}{k},\frac{n}{k}\right)^{-1} \phi_t^{-1}\left(1+\frac{|u_t|^k}{\left((n+1)/k\right)\left(1+sign(u_t)\lambda\right)^k \phi_t^k}\right)^{-\frac{n+1}{k}}, \tag{2}$$

where

$$u_t \equiv r_t - m_t = r_t - \left(c\sigma_t + a + b r_{t-1}\right) \tag{3}$$

are deviations of returns r_t from their conditional mode m_t (in the case of the symmetric GT the mean and the mode are equal). The scaling parameter ϕ_t is a time-varying dispersion measure related to σ_t when it exists, k and n are positive kurtosis parameters controlling respectively the peakness around the mode and the tails of the distribution, λ is a skewness parameter with domain the open interval (–1, 1), $sign(u_t)$ is the sign function (i.e., sign(u_t)= –1 for $u_t \leq 0$ and 1 for $u_t > 0$) and $B(w, z) = \Gamma(w)\,\Gamma(z) / \Gamma(w+z)$ is the beta function. Values of $k < 2$ are associated with leptokurtic (peaked) distributions relative to the normal distribution and smaller values of n with fat tailed distributions.

The *SGT* includes the *GT* of McDonald and Newey (1988) for $\lambda = 0$, the skewed t of Hansen (1994) for $k = 2$, the student t for $\lambda = 0$ and $k = 2$ and the Cauchy for $\lambda = 0$, $k = 2$ and $n = 1$. For $n \rightarrow \infty$, it yields the skewed *GED* of Theodossiou (2000), which includes the *GED* for $\lambda = 0$, the Laplace or double exponential for $\lambda = 0$ and $k = 1$, the normal for $\lambda = 0$ and $k = 2$ and the uniform for $\lambda = 0$ and $k \rightarrow \infty$.

When $n > 2$, the conditional mean and variance of r_t, see eq. (A9) and (A11) in the Appendix, are:

$$\mu_t = E\left(r_t \,|\, I_{t-1}\right) = m_t + E\left(u_t|I_{t-1}\right) = m_t + p\sigma_t \tag{4}$$

and

$$\sigma_t^2 = \mathrm{var}\left(u_t \,|\, I_{t-1}\right) = \left(A_2 - A_1^2\right)\phi_t^2\,, \tag{5}$$

where

$$p = A_1 \Big/ \sqrt{A_2 - A_1^2}\ , \tag{6}$$

$$A_1 = 2\lambda \left(\frac{n+1}{k}\right)^{\frac{1}{k}} B\left(\frac{2}{k}, \frac{n-1}{k}\right) B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1} \tag{7}$$

and

$$A_2 = \left(1+3\lambda^2\right)\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{3}{k}, \frac{n-2}{k}\right) B\left(\frac{1}{k}, \frac{n}{k}\right)^{-1}. \tag{8}$$

It follows easily from eq. (4) that the parameter $p = (\mu_t - m_t) / \sigma_t$. This measure, known as the Pearson's skewness, is a symmetric function of the skewness parameter λ and a highly non-linear function of the kurtosis parameters k and n. This is a key measure for the issues investigated in this paper.

Figure 1 provides a graphical illustration of the parameter p for various values of λ, k and n. It is clear from the figure that as λ increases in magnitude, p also increases in magnitude. Negative values of λ are associated with negative values of p and vice versa. For λ =0, p =0. Interestingly, larger values of k and n are also associated with larger values of p. Clearly, p is a monotonic function of λ, k and n.

Intertemporal Pricing Model

The substitution of $m_t \equiv c\sigma_t + a + b\,r_{t-1}$ into the conditional mean excess return eq. (4) gives

$$\mu_t = c\sigma_t + p\sigma_t + a + b\,r_{t-1}, \tag{9}$$

where $c\ \sigma_t$ is the "pure" risk premium, which is expected to be positive and $p\ \sigma_t$ is the skewness-kurtosis premium. The latter, depending on the direction of skewness in the distribution of excess returns, can be negative, zero or positive.

The regression in (1) can be written in the following equivalent form

$$r_t = \mu_t + \varepsilon_t = \left(c+p\right)\sigma_t + a + b\,r_{t-1} + \varepsilon_t. \tag{10}$$

Unlike u_t, the error term $\varepsilon_t = u_t - p\ \sigma_t$ has a zero expected value. Note that the term $(c + p)\ \sigma_t = \xi\ \sigma_t$ measures the combined impact of "pure" and skewness-kurtosis risk on the mean of a portfolio's excess returns. This equation provides the foundation for exploring and explaining the contradictory findings in the literature regarding the risk-return relationship. In case of negative skewness, depending on the size of p, the value of ξ can be positive, zero or negative.

Conditional Variance

The conditional variance of excess returns is specified as a function of the past regression errors, including their squared, absolute and standardized values and past conditional variances. The following four popular GARCH models are considered:

GARCH of Bollerslev (1986): $\sigma_t^2 = v + \beta\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2$ (11a)

GJR-GARCH of Glosten, et al. (1993): $\sigma_t^2 = v + \left(\delta N_{t-1} + \beta\right)\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2$ (11b)

QGARCH of Sentana (1995): $\sigma_t^2 = v + \delta\varepsilon_{t-1} + \beta\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2$ (11c)

EGARCH of Nelson (1991): $\ln\sigma_t^2 = v + \delta z_{t-1} + \beta g\left(z_{t-1}\right) + \gamma\ln\sigma_{t-1}^2$ (11d)

where $N_t = 0$ for $\varepsilon_t \geq 0$ and $N_t = 1$ for $\varepsilon_t < 0$ and $g(z_t) = |z_t| - E|z_t|$ and $z_t = \varepsilon_t / \sigma_t$.

In eq. (11b)-(11d), the parameter δ captures asymmetric volatility. In eq. (11b), δ is expected to be positive and in (11c) and (11d) negative. Such values would imply that volatility is higher in stock market downturns than in upturns. This kind of asymmetry is typically an indication of negative skewness in the distribution of excess returns.

Estimation

Parameter estimates for eq. (1) are obtained via numerical optimization of the sample log-likelihood

$$\max_{\theta} L\left(\theta\right) = \sum_{t=1}^{T}\log f\left(r_t \middle| \theta, I_{t-1}\right), \quad (12)$$

where f is a conditional probability density function for r_t and θ = [c, a, b, v, δ, β, γ, λ, k, n]'. For eq. (11a), $\delta = 0$. The t-values for the estimators are computed using robust standard errors. Moreover, the parameter p is endogenously determined using the MLE estimators for λ, k and n along with eq. (6)–(8).

At this point, it is important to note that in the presence of skewness, the estimation of eq. (1) using a symmetric log-likelihood specification will result in a biased estimator for the price of “pure” risk or the GARCH-in-mean effect, measured by c. In fact, the resulting estimator will be $\xi = c + p$ and not c. Moreover, the computed standardized errors will not possess a zero mean and a unit variance and the conditional variances will be misspecified.

3. Monte Carlo Simulations

Random Samples

For the simulations 1,000 samples per GARCH model are used. Each sample includes 1,052 randomly generated returns using similar parameters to those of the monthly models estimated in the next section. The returns are generated as follows:

1. A vector z= [z_0,z_1,..,z_T]' of 1,053 standardized random errors is drawn from the SGT distribution below:

$$z \sim SGT\left(\mu = 0, \sigma = 1, \lambda = -0.285, k = 2, n = 10\right),$$

with Pearson's skewness p = –0.43, standardized skewness SK = –0.656 and standardized kurtosis KU = 4.428. The latter values are computed using eq. (6), (A15) and (A18), respectively.

2. The arithmetic mean and standard deviation of monthly excess returns are used as starting values for σ_t μ_t, ε_t and r_t. That is, σ_0 = 5.164, μ_0 = 0.502, $\varepsilon_0 = 5.164\, z_0$ and $r_0 = 0.502 + \varepsilon_0$.

3. Random monthly excess returns for each of the four GARCH specifications are generated using the recursive equations below:

$$\text{GARCH: } \sigma_t^2 = 0.8 + 0.12\,\varepsilon_{t-1}^2 + 0.85\,\sigma_{t-1}^2$$

$$\text{GJR-GARCH: } \sigma_t^2 = 0.8 + \left(0.15 N_{t-1} + 0.04\right)\varepsilon_{t-1}^2 + 0.85\,\sigma_{t-1}^2$$

$$\text{QGARCH: } \sigma_t^2 = 0.8 - 0.5\,\varepsilon_{t-1} + 0.12\,\varepsilon_{t-1}^2 + 0.85\,\sigma_{t-1}^2$$

$$\text{EGARCH: } \ln\sigma_t^2 = 0.17 - 0.09\,z_{t-1} + 0.22\left(|\,z_{t-1}\,| - E\,|\,z_{t-1}\,|\right) + 0.95\ln\sigma_{t-1}^2$$

$$\varepsilon_t = z_t \sigma_t$$

$$r_t = 0.488 + 0.025\, r_{t-1} + \varepsilon_t\,,$$

where N_{t-1} = 1 for $\varepsilon_{t-1} < 0$ and zero otherwise and t = 1, 2, …, 1,052. Note that the parameter ξ in the return equation above is set to zero, i.e., $c = -p = 0.43$.

Simulation Results

Eq. (1) for each GARCH model is estimated 1,000 times using the randomly generated samples of excess returns and the log-likelihood specifications of a) *SGT*, b) *GT*, c) *GED*, d) *T* and e) *N*.

The first panel of Table 1 reports the arithmetic mean and standard deviation of the estimated values of the "pure" price of risk (parameter c) and the skewness-kurtosis price of risk (parameter p) for each of the four SGT-GARCH specifications in the 1,000 random samples and the percentage of times that estimates of c, p and ξ are statistically different from their theoretical values. The remaining panels report similar statistics for the (contaminated) pure price risk, c, of the four GARCH specifications under the symmetric log-likelihood specifications of *GT, GED*, *T* and *N*.

Interestingly, in the case of the *SGT* distribution, the simulated values of the parameters c and p are extremely close to their respective theoretical values of 0.43 and −0.43 for all four GARCH models. The *SGT* simulation results uncover fully the hypothesized positive relationship between risk and return. This is, however, not the case with the estimated values of c based on the *GT*, *GED*, *T* and *N* log-likelihood specifications, where the estimated values of c are close to the theoretical value of $\xi = c + p = 0$.

Generally, these simulation results indicate that, depending on the extent of negative skewness in the distribution of excess returns, the estimation of intertemporal pricing models using symmetric log-likelihood specifications may yield conflicting findings regarding the risk-return relationship.

4. Empirical Findings

Preliminary Statistics

The data includes daily, weekly and monthly value-weighted excess returns over the one-month Treasury bill rate of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. It covers the period July 1, 1926 to February 28, 2014 and is obtained from French's website.[3] It is transformed into continuously compounded returns using the equation $r_t = 100 * \log\left(1 + y_t/100\right)$, where y_t is a geometric return expressed as a percentage and log is the natural logarithm.

[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html.

Preliminary statistics for the daily, weekly and monthly frequencies of CRSP excess returns are reported on Table 2. It is worth noting that all three frequencies exhibit negative skewness and standardized kurtosis ranging between 6.1 and 7, which is at least twice that of the normal distribution, which is equal to 3. These findings along with the KS statistics indicate serious departures of the data from normality.[4]

Main Results

Table 3, 4 and 5 present the parameter estimates of the intertemporal excess return equation in (1) for all four GARCH models under the *SGT* log-likelihood specification based on daily, weekly and monthly data. The results are quite similar for all GARCH models and frequencies. In all cases the estimated values for the pure price of risk coefficient, c, are positive and statistically significant. On the other hand, the estimated values for the skewness-kurtosis price of risk, p, are negative and statistically significant. The combined impact of the two, measured by $\xi = c + p$, is close to zero and statistically insignificant in all cases. Interestingly, the asymmetric volatility coefficient, δ, is statistically significant and consistent in all three cases with the presence of asymmetric volatility.

Despite the fact that the form of negative skewness implied by asymmetric volatility is factored out to some extent by the conditional variance equations, still the results indicate that the main force driving the mixed findings in the literature regarding the risk-return relationship is skewness in the return distribution coupled with fact that the estimation of the relevant pricing models is based on symmetric log-likelihood specifications.

Replication of Previous Studies

To investigate further the conflicting findings in the literature, the results of several important studies related to the US are replicated using the same data frequencies and periods and similar GARCH and log-likelihood specifications. The replications are performed for studies comparable to the framework employed in this paper.

[4] Data are Winsorized to plus/minus four standard deviations from their sample means.

The results regarding the estimated combined risk-skewness effect (parameter $\xi = c + p$), presented in panel A of Table 6, are qualitatively similar to those found in the original studies. There are, however, some minor differences which may be attributed to the omission of explanatory variables, which could not be replicated and the use of robust standard errors for the estimators, which may not have been used in previous studies.

The results for the *SGT* specifications, presented on Panel B of Table 6, indicate a positive "pure" risk premium and a negative skewness-kurtosis premium. In both cases, the relevant coefficients are statistically significant at the 1% level, while their combined impact, measured by $\xi = c + p$, is in line with that of panel A. Once more, the results point out to skewness as being the reason for the conflicting findings in the literature on the risk-return relation.[5]

Other Risk-Return Specifications

The investigation into the risk-return relationship is continued by augmenting eq. (1) with an extra term for the lag-value of the conditional standard deviation of returns, i.e.

$$r_t = c\sigma_t + d\sigma_{t-1} + a + b\,r_{t-1} + u_t\,. \tag{13}$$

The estimated parameters for the three data frequencies and the four GARCH models using an *SGT* log-likelihood specification are presented on Table 7. Notice that the estimated values of d are quite mixed in terms of their signs and statistical significance. These results are likely to be triggered by the strong correlation between σ_t and σ_{t-1}, which ranges from 0.95 to 0.99. Note, however, that the estimated values of the skewness-kurtosis coefficient p and the combined impact $\xi = c + d + p$ are almost identical to those on Tables 3, 4 and 5.

The next step was to run the model with the lag value of σ_t, only. The results, which are available upon request, are almost identical to those of the aforementioned tables.

Eq. (1) is further augmented with the inclusion of the conditional variance, i.e.

$$r_t = c\sigma_t + d\sigma_t^2 + a + b\,r_{t-1} + u_t\,. \tag{14}$$

[5] The replication was also performed on the entire dataset. The results, however, are qualitatively similar.

The estimated parameters are presented on Table 8. Note that in almost every case the estimated values of d are positive and statistically insignificant. Nevertheless, the estimated values of p are in line with the previous results.

Prices of Risk across Data Frequencies

The stochastic behavior of pure and skewness-kurtosis prices of risk across data frequencies requires further investigation. For this purpose, the pricing model of eq. (9) is rewritten as

$$\mu_{\Delta t,t} = \left(c_{\Delta t} + p_{\Delta t}\right)\sigma_{\Delta t,t} + a_{\Delta t} + b_{\Delta t}\, r_{\Delta t,t-1}, \tag{15}$$

where Δt denotes the time length between two consecutive excess returns. For example, for daily, weekly and monthly returns Δt = 1/252, 5/252 and 21/252, respectively, where 252 is for the trading days in each year. Clearly, all parameters in the above conditional mean excess return equation, including those for the conditional standard deviation, depend on the data frequency indicated by Δt.

Eq. (15) is also written as

$$\mu^*_{\Delta t,t} = \frac{\left(c_{\Delta t} + p_{\Delta t}\right)}{\sqrt{\Delta t}}\,\sigma^*_{\Delta t,t} + \frac{a_{\Delta t}}{\Delta t} + \frac{b_{\Delta t}}{\Delta t} r_{\Delta t,t-1}, \tag{16}$$

where $\mu^*_{\Delta t,t} = \mu_{\Delta t,t}/\Delta t$ and $\sigma^*_{\Delta t,t} = \sigma_{\Delta t,t}/\sqrt{\Delta t}$ are annualized measures of the conditional mean and conditional standard deviation of excess returns for each data frequency. Moreover, $\mu^*_{\Delta t} \equiv E\left(\mu^*_{\Delta t,t}\right)$ $= \mu_{\Delta t}/\Delta t$ and $\sigma^*_{\Delta t} \equiv E\left(\sigma^*_{\Delta t,t}\right) = \sigma_{\Delta t}/\sqrt{\Delta t}$, where $\mu_{\Delta t}$ and $\sigma_{\Delta t}$ are respectively the unconditional expected values of $\mu_{\Delta t,t}$ and $\sigma_{\Delta t,t}$. Under normality, or more generally when returns are Levy processes, $E\left(\mu^*_{\Delta t,t}\right) = \mu^*$ and $E\left(\sigma^*_{\Delta t,t}\right) = \sigma^*$, regardless of Δt. In general, departures from these equalities occur when higher order moment dependencies (non-linearities), including skewness and kurtosis, are present in the return series.

It follows easily from eq. (16), under standard regularity conditions, that

$$\mu^*_{\Delta t} = \frac{\left(c_{\Delta t} + p_{\Delta t}\right)}{\left(1 - b_{\Delta t}\right)\sqrt{\Delta t}}\,\sigma^*_{\Delta t} + \frac{a_{\Delta t}}{\left(1 - b_{\Delta t}\right)\Delta t}, \tag{17}$$

where

$$c^*_{\Delta t} \equiv c_{\Delta t} \Big/ \left((1 - b_{\Delta t}) \sqrt{\Delta t} \right),$$

$$p^*_{\Delta t} \equiv p_{\Delta t} \Big/ \left((1 - b_{\Delta t}) \sqrt{\Delta t} \right)$$

and

$$\xi^*_{\Delta t} = c^*_{\Delta t} + p^*_{\Delta t}$$

are annualized measures for the pure and skewness-kurtosis prices of risk.

Figure 2 presents the estimated values of the above annualized measures for the GJR model and data frequencies ranging from one to twenty-one trading days. For all data frequencies, the pure price of risk $c^*_{\Delta t}$ is positive, the skewness-kurtosis price of risk $p^*_{\Delta t}$ negative and their combined value $\xi^*_{\Delta t}$ close to zero. Figure 3 presents their t-values, denoted by t_c, t_p and t_ξ, respectively. All t_c and t_p values lie outside the interval ±1.96, therefore, $c^*_{\Delta t}$ and $p^*_{\Delta t}$ are statistically significant. On the contrary, the t_ξ values lie inside the interval indicating that $\xi^*_{\Delta t}$ is statistically insignificant across all data frequencies considered. Interestingly, as Δt gets larger, $c^*_{\Delta t}$ and $p^*_{\Delta t}$ decrease in magnitude and at the monthly frequency radiates around ±1.6. The fact that these estimates are larger in higher data frequencies may be attributed to the presence of strong moment dependencies, which become weaker in lower data frequencies. For the remaining GARCH models, the results are remarkably similar.

For each data frequency, the time-series behavior of λ (main determinant of $p^*_{\Delta t}$, see eq. 6), is investigated using Hansen's (1994) specification

$$\lambda_t = \lambda_0 + \lambda_1 \varepsilon_{t-1} + \lambda_2 \varepsilon^2_{t-1}\,. \tag{18}$$

The estimates for λ_1 range between –0.0117 and 0.0104 and for λ_2 between 0.0001 and 0.0061. These estimates are statistically insignificant, except for that of λ_2 at the daily data frequency. Similar findings are documented for the remaining two determinants of $p^*_{\Delta t}$, i.e., the kurtosis parameters k and n. These results indicate that $p^*_{\Delta t}$ is not time-varying within each data frequency.

The results in this sub-section are consistent with the hypothesis that because investors hedge constantly against fluctuations in the distribution of stock returns, higher moments, including skewness and kurtosis, are priced more in the short- than in the long-term.

5. Summary and Conclusions

The theoretical analysis carried out using an analytical framework based on the popular *SGT* distribution indicates that the conditional mean of a portfolio's excess returns is a function of a "pure" risk premium, which is expected to be positive and a skewness-kurtosis premium which has the same sign as skewness. Depending on the extent of negative skewness, the combined size of the latter premia can be positive, zero or negative.

Estimation of intertemporal pricing models using log-likelihood specifications based on symmetric probability distributions results in price of risk measures contaminated by that of the skewness-kurtosis price of risk. This is in fact the main reason behind the contradictory findings in the finance literature regarding the risk-return relationship. The latter finding is confirmed via Monte Carlo simulations.

Estimation of standard intertemporal pricing models based on a *SGT* log-likelihood specification using daily, weekly and monthly CRSP excess returns over a long period, confirms the presence of a positive risk premium and a negative skewness-kurtosis premium with zero combined impact. Replication of the results of previous studies using the same data frequencies and periods and model specifications yields mixed results when estimated using symmetric log-likelihood specification and similar results, as the ones presented previously, when estimated using a skewed log-likelihood specification.

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Appendix –Mean, Variance, Skewness and Kurtosis of the *SGT*

The j^{th} non-centered moment, for $0 \le j < n$, is

$$M_j = \int_{-\infty}^{\infty} u^j f\, du = (-1)^j C\int_0^{\infty} u^j \left(1+\frac{u^k}{((n+1)/k)(1-\lambda)^k \phi^k}\right)^{-\frac{n+1}{k}} du$$

$$+C\int_0^{\infty} u^j \left(1+\frac{u^k}{((n+1)/k)(1+\lambda)^k \phi^k}\right)^{-\frac{n+1}{k}} du\,, \qquad \text{(A1)}$$

where $u = r - m$, k, $n > 0$ and $-1 < \lambda < 1$. The substitution of

$$u = (1\pm\lambda)\phi((n+1)/k)^{\frac{1}{k}} t^{\frac{1}{k}} (1-t)^{-\frac{1}{k}}\,, \qquad \text{(A2)}$$

and

$$u^j du = (1\pm\lambda)^{j+1} \phi^{j+1} \frac{1}{k}((n+1)/k)^{\frac{j+1}{k}} t^{\frac{j+1}{k}-1} (1-t)^{-\frac{j+1}{k}-1} dt \qquad \text{(A3)}$$

into M_j gives

$$M_j = C\left[(-1)^j (1-\lambda)^{j+1} + (1-\lambda)^{j+1}\right]\phi^{j+1}\frac{1}{k}((n+1)/k)^{\frac{j+1}{k}} \int_0^1 t^{\frac{j+1}{k}-1}(1-t)^{\frac{n-j}{k}-1} dt\,,$$

$$= C\left[(-1)^j (1-\lambda)^{j+1} + (1-\lambda)^{j+1}\right]\phi^{j+1}\frac{1}{k}((n+1)/k)^{\frac{j+1}{k}} B\left(\frac{j+1}{k},\frac{n-j}{k}\right), \qquad \text{(A4)}$$

where $B\left(\frac{j+1}{k},\frac{n-j}{k}\right) \equiv \int_0^{\infty} t^{\frac{j+1}{k}-1}(1-t)^{\frac{n-j}{k}-1} dt$.

For f to be a proper p.d.f., $M_0 = 1$, and

$$C = 0.5k\left(\frac{n+1}{k}\right)^{-\frac{1}{k}} B\left(\frac{1}{k},\frac{n}{k}\right)^{-1} \phi^{-1}. \qquad \text{(A5)}$$

The substitution of (A5) into (A4), gives

$$M_j = A_j \phi^j. \qquad \text{(A6)}$$

where

$$A_j = \frac{1}{2}\left[(-1)^j (1-\lambda)^{j+1} + (1+\lambda)^{j+1}\right] G_j \qquad \text{(A7)}$$

and

$$G_j = \left(\frac{n+1}{k}\right)^{\frac{j}{k}} B\left(\frac{j+1}{k},\frac{n-j}{k}\right) B\left(\frac{1}{k},\frac{n}{k}\right)^{-1}, \text{ for } j = 1, 2, \ldots, < n. \qquad \text{(A8)}$$

Expected Value of *u*

For $j = 1$ and $n > 1$,

$$M_1 = Eu = A_1\phi, \tag{A9}$$

where

$$A_1 = \frac{1}{2}\left[(-1)(1-\lambda)^2 + (1+\lambda)^2\right]G_1 = 2\lambda G_1.$$

Variance

For $j = 2$ and $n > 2$,

$$M_2 = Eu^2 = A_2\phi^2, \tag{A10}$$

where

$$A_2 = \frac{1}{2}\left[(-1)(1-\lambda)^3 + (1+\lambda)^3\right]G_2 = \left(1+3\lambda^2\right)G_2.$$

The variance of u is

$$\sigma^2 = \text{var}(u) = M_2 - M_1^2 = \left(A_2 - A_1^2\right)\phi^2, \tag{A11}$$

where $A_2 - A_1^2 > 0$. Substitution of $\phi = \sigma\big/\sqrt{A_2 - A_1^2}$ into (A9) gives

$$M_1 = Eu = \left(A_1\big/\sqrt{A_2 - A_1^2}\right)\sigma = p\sigma. \tag{A12}$$

Third Centered Moment and Skewness

For $j = 3$ and $n > 3$,

$$M_3 = Eu^3 = A_3\phi^3, \tag{A13}$$

where

$$A_3 = \frac{1}{2}\left[(-1)(1-\lambda)^4 + (1+\lambda)^4\right]G_3 = 4\lambda\left(1+\lambda^2\right)G_3.$$

The third centered moment of u is

$$E(u-Eu)^3 = Eu^3 - 3Eu^2Eu + 2(Eu)^3 = \left(A_3 - 3A_2A_1 + 2A_1^3\right)\phi^3. \tag{A14}$$

The standardized skewness is

$$SK = \frac{E(u-Eu)^3}{\sigma^3} = \frac{\left(A_3 - 3A_2A_1 + 2A_1^3\right)}{\left(A_2 - A_1^2\right)^{3/2}}. \tag{A15}$$

Fourth Centered Moment and Kurtosis

For $j = 4$ and $n > 4$,

$$M_4 = Eu^4 = A_4\phi^4, \tag{A16}$$

where

$$A_4 = \frac{1}{2}\left[(-1)(1-\lambda)^5 + (1+\lambda)^5\right]G_4 = \left(1+10\lambda^2+5\lambda^4\right)G_4 .$$

The fourth centered moment of *u* is

$$E\left(u-Eu\right)^4 = Eu^4 - 4Eu^3Eu + 6Eu^2\left(Eu\right)^2 - 3\left(Eu\right)^4$$

$$= \left(A_4 - 4A_3A_1 + 6A_2A_1^2 - 3A_1^4\right)\phi^4. \qquad \text{(A17)}$$

The standardized kurtosis is

$$KU = \frac{E\left(u-Eu\right)^4}{\sigma^4} = \frac{\left(A_4 - 4A_3A_1 + 6A_2A_1^2 - 3A_1^4\right)}{\left(A_2 - A_1^2\right)^2}. \qquad \text{(A18)}$$

Table 1. Monte Carlo Simulations of the Intertemporal Asset Pricing Model

Distribution	Statistics	GARCH-M	GJR-GARCH-M	QGARCH-M	EGARCH-M
A. *SGT* - Skewed Generalized t					
c	Mean	0.430	0.435	0.430	0.435
	Std	0.069	0.071	0.072	0.076
	Reject rate, %	5.9	4.2	5.4	5.1
p	Mean	-0.431	-0.434	-0.433	-0.435
	Std	0.061	0.064	0.063	0.066
	Reject rate, %	5.4	5.4	5.3	4.6
$\xi = c + p$	Mean	-0.001	0.001	-0.003	-0.0002
	Std	0.035	0.034	0.036	0.0401
	Reject rate, %	0.12	0	0	0
B. *GT* - Generalized t					
c	Mean	0.035	0.034	0.025	0.032
	Std	0.045	0.048	0.058	0.052
	Reject rate, %	100	100	100	100
C. *GED* - Generalized Error Distribution					
c	Mean	0.042	0.045	0.036	0.043
	Std	0.050	0.050	0.063	0.056
	Reject rate, %	100	100	100	100
D. *T* - Student t					
c	Mean	0.044	0.044	0.035	0.041
	Std	0.045	0.046	0.058	0.051
	Reject rate, %	100	100	100	100
E. *N* - Normal					
c	Mean	-0.012	-0.001	-0.017	-0.002
	Std	0.053	0.049	0.062	0.070
	Reject rate, %	100	100	100	99.8

Notes:

Simulations are based on 1,000 random samples with theoretical values for c = 0.43 and p = -0.43.

Mean and Std are the simple arithmetic mean and standard deviation of the estimates in the random samples.

Reject rate is the percent of estimates which are statistically different from their theoretical values.

Table 2. Preliminary Statistics for CRSP Excess Returns

Statistics	Daily	Weekly	Monthly
Mean	0.024	0.117	0.502
Std	0.990	2.362	5.164
Skewness	-0.198	-0.365	-0.599
Kurtosis	7.060	6.248	6.122
KS	0.096	0.180	0.384
OBS	23,173	4,574	1,052

Notes:

Data covers the period July 1, 1926 - February 28, 2014.

Excess returns are continuously compounded.

Skewness is equal to $m_3 / m_2^{3/2}$ and kurtosis m_4 / m_2^2, where m_j is the sample estimate for the jth moment around the mean.

KS is the Kolmogorov-Smirnov statistic.

Table 3. Excess Return Pricing Model Estimation Using Daily Data

Parameters	GARCH-M	GJR-GARCH-M	QGARCH-M	EGARCH-M
A. Conditional mean				
a	0.035	0.032	0.027*	0.043
	(0.012)	(0.011)	(0.012)	(0.011)
b	0.116	0.127	0.126	0.124
	(0.007)	(0.007)	(0.007)	(0.007)
c	0.148	0.140	0.141	0.122
	(0.022)	(0.021)	(0.022)	(0.021)
B. Skewness price of risk				
p	-0.136	-0.142	-0.140	-0.144
	[-0.158 -0.114]	[-0.165 -0.121]	[-0.161 -0.121]	[-0.163 -0.122]
$\xi = c + p$	0.012#	0.002#	0.001#	-0.022#
	(0.018)	(0.017)	(0.018)	(0.017)
C. Conditional variance				
v	0.006	0.007	0.012	-0.006
	(0.001)	(0.001)	(0.001)	(0.001)
β	0.084	0.031	0.085	0.152
	(0.005)	(0.003)	(0.005)	(0.008)
δ	-	0.101	-0.067	-0.081
		(0.009)	(0.006)	(0.005)
γ	0.910	0.910	0.902	0.988
	(0.006)	(0.006)	(0.006)	(0.002)
D. Unconditional mean and standard deviation				
μ	0.053	0.039	0.032	0.030
σ	1.000	0.907	0.961	0.779
E. Skewness and kurtosis parameters				
λ	-0.090	-0.094	-0.093	-0.095
	(0.008)	(0.008)	(0.008)	(0.008)
n	12.522	12.070	11.663	11.285
k	1.632	1.683	1.697	1.706
F. Other statistics				
$L(\theta)$	-27,665.9	-26,902.7	-26,889.4	-26,862.4
SK	-0.261	-0.265	-0.262	-0.269
KU	4.561	4.473	4.493	4.522
OBS	23,173	23,173	23,173	23,173

Notes:

Estimation of eq. 1 under the SGT log-likelihood specification for each GARCH model.

All coefficients are significant at the 1% level, unless otherwise noted.

*, ** Statistically significant at the 5% and 10%, respectively. # statistically insignificant.

Parentheses include the standard errors and brackets the confidence intervals for the estimators

Confidence Intervals for p are based on Rapach and Wohar (2009) bootstrapping procedure

$L(\theta)$ is the log-likelihood value.

SK and KU are standardized skewness and kurtosis, computed using eq. (A14) and (A18)

Computation of unconditional variance of excess returns

GARCH & QGARCH	GJR-GARCH	EGARCH
$\sigma^2 = v/(1-\beta-\gamma)$	$\sigma^2 = v/(1-0.5\delta-\beta-\gamma)$	$\sigma^2 \approx \exp(v/(1-\gamma))$

Computation of unconditional mean return $\mu = (\alpha + \xi\sigma)/(1-b)$

c is the pure price of risk and p the skewness-kurtosis price of risk.

Table 4. Excess Return Pricing Model Estimation Using Weekly Data

Parameters	GARCH-M	GJR-GARCH-M	QGARCH-M	EGARCH-M
A. Conditional mean				
a	0.131#	0.168*	0.149**	0.179*
	(0.086)	(0.085)	(0.088)	(0.084)
b	-0.002#	0.018#	0.018#	0.017#
	(0.016)	(0.016)	(0.016)	(0.016)
c	0.397	0.356	0.360	0.351
	(0.060)	(0.059)	(0.061)	(0.058)
B. Skewness price of risk				
p	-0.365	-0.365	-0.364	-0.369
	[-0.418 -0.316]	[-0.416 -0.315]	[-0.419 -0.312]	[-0.418 -0.319]
$\xi = c + p$	0.032#	-0.009#	-0.004#	-0.018#
	(0.048)	(0.047)	(0.050)	(0.045)
C. Conditional variance				
v	0.072	0.102	0.162	0.034
	(0.019)	(0.027)	(0.043)	(0.009)
β	0.097	0.040	0.113	0.185
	(0.013)	(0.009)	(0.016)	(0.020)
δ	-	0.110	-0.200	-0.080
		(0.024)	(0.045)	(0.012)
γ	0.889	0.880	0.855	0.978
	(0.015)	(0.019)	(0.022)	(0.006)
D. Unconditional mean and standard deviation				
σ	2.268	2.020	2.250	2.166
μ	0.203	0.153	0.143	0.142
E. Skewness and kurtosis parameters				
λ	-0.240	-0.239	-0.238	-0.242
	(0.022)	(0.022)	(0.022)	(0.022)
n	8.937	8.088	7.775	8.038
k	2.057	2.196	2.244	2.225
F. Other statistics				
$L(\theta)$	-9,543.6	-9,508	-9,502.4	-9,492.5
SK	-0.574	-0.560	-0.559	-0.558
KU	4.450	4.443	4.478	4.413
OBS	4,574	4,574	4,574	4,574

Notes:

See Table 3

Table 5. Excess Return Pricing Model Estimation Using Monthly Data

Parameters	GARCH-M	GJR-GARCH-M	QGARCH-M	EGARCH-M
A. Conditional mean				
a	0.545#	0.702#	0.618#	0.605#
	(0.488)	(0.519)	(0.532)	(0.464)
b	-0.001#	0.024#	0.026#	0.016#
	(0.030)	(0.034)	(0.033)	(0.034)
c	0.480	0.402	0.420	0.433
	(0.125)	(0.134)	(0.137)	(0.129)
B. Skewness price of risk				
p	-0.444	-0.423	-0.427	-0.434
	[-0.572 -0.313]	[-0.534 -0.309]	[-0.519 -0.336]	[-0.552 -0.315]
$\xi = c + p$	0.036#	-0.021#	-0.007#	-0.003#
	(0.112)	(0.115)	(0.132)	(0.094)
C. Conditional variance				
v	0.842	1.275*	1.500*	0.169
	(0.274)	(0.512)	(0.615)	(0.048)
β	0.115	0.042**	0.118	0.222
	(0.020)	(0.026)**	(0.023)	(0.046)
δ	-	0.132*	-0.484*	-0.088
		(0.063)	(0.197)	(0.026)
γ	0.847	0.829	0.817	0.946
	(0.025)	(0.039)	(0.041)	(0.016)
D. Unconditional mean and standard deviation				
σ	4.707	4.499	4.804	4.782
μ	0.714	0.622	0.600	0.600
E. Skewness and kurtosis parameters				
λ	-0.293	-0.279	-0.280	-0.285
	(0.042)	(0.043)	(0.043)	(0.048)
n	10.473	9.000	10.115	9.788
k	2.031	2.144	2.104	2.111
F. Other statistics				
$L(\theta)$	-3,061.7	-3,056.7	-3,055.1	-3,053.8
SK	-0.646	-0.622	-0.603	-0.619
KU	4.284	4.353	4.173	4.237
OBS	1,052	1,052	1,052	1,052

Notes:

See Table 3.

Table 6. Replication of the Results of Previous Studies

Model and Author(s)	*Pdf*	Dataset, span and frequency			A. Original Specification Risk-return relation found	$\xi = c + p$	B. SGT Specification $\xi = c + p$	c	p
1. GARCH-M									
French et. al. (1987)	*N*	CRSP	1/1928 - 12/1984	Monthly	Signif. Posit.	0.065#	0.028#	0.407*	-0.379
						(0.119)	(0.143)	(0.160)	[-0.431 -0.323]
Closten et. al. (1993)	*N*	CRSP	4/1951 - 12/1989	Monthly	Insign. Posit.	0.064#	0.052#	0.447*	-0.395
						(0.622)	(0.181)	(0.183)	[-0.447 -0.338]
Baillie and DeGennaro (1990)	*N*	CRSP	1/1970 - 12/1987	Daily	Signif. Posit.	0.108**	0.079#	0.131*	-0.052
						(0.061)	(0.059)	(0.065)	[-0.076 -0.029]
Baillie and DeGennaro (1990)	*T*	CRSP	1/1970 - 12/1987	Daily	Insign. Posit.	0.084#	0.079#	0.131*	-0.052
						(0.058)	(0.059)	(0.065)	[-0.076 -0.029]
Theodossiou and Lee (1995)	*N*	S&P500	1/1976 - 12/1991	Weekly	Insign. Posit.	0.165#	0.226#	0.451	-0.225#
						(1.564)	(0.383)	(0.140)	[-0.268 -0.171]
Bansal and Lundblad (2002)	*N*	DataStream	1/1973 - 12/1998	Monthly	Insign. Posit.	1.114#	1.204#	1.511#	-0.307
						(0.753)	(1.283)	(1.297)	[-0.388 -0.222]
Lanne and Saikkonen (2007)	*T*	S&P500	1/1946 - 12/2002	Monthly	Signif. Posit.	0.307*	0.286*	0.713*	-0.427
						(0.143)	(0.130)	(0.324)	[-0.485 -0.367]
Lundblad (2007)	*N*	CRSP†	1/1836 - 12/2003	Monthly	Signif. Posit.	0.197*	0.191*	0.486	-0.295
						(0.103)	(0.095)	(0.120)	[-0.359 -0.228]
2. GJR GARCH-M									
Closten et. al. (1993)	*N*	CRSP	4/1951 - 12/1989	Monthly	Signif. Neg.	-0.119*	-0.118*	0.084#	-0.202
						(0.028)	(0.189)	(0.202)	[-0.293 -0.128]
Lundblad (2007)	*N*	CRSP†	1/1836 - 12/2003	Monthly	Signif. Posit.	0.144**	0.142**	0.429	-0.287
						(0.081)	(0.081)	(0.122)	[-0.352 -0.219]
3. QGARCH-M									
Campbell and Hentschel (1992)	*N*	CRSP	1/1926 - 12/1988	Daily	Insign. Posit.	0.035#	-0.010#	0.127	-0.137
						(0.025)	(0.020)	(0.025)	[-0.174 -0.112]
Lundblad (2007)	*N*	CRSP	1/1836 - 12/2003	Monthly	Signif. Posit.	0.138*	0.132*	0.419	-0.287
						(0.061)	(0.063)	(0.108)	[-0.371 -0.221]
4. EGARCH-M									
Nelson (1991)	*GED*	CRSP	7/1962 - 12/1987	Daily	Insign. Neg.	-0.005#	-0.011#	0.084*	-0.095
						(0.041)	(0.035)	(0.043)	[-0.118 -0.073]
Lundblad (2007)	*N*	CRSP†	1/1836 - 12/2003	Monthly	Signif. Posit.	0.158**	0.140**	0.433	-0.293
						(0.096)	(0.841)	(0.133)	[-0.358 -0.227]

Notes:

†Data for the years 1836-1925 are compiled by Schwert. All coefficients are significant at the 1% level, unless otherwise noted.

*, ** Statistically significant at the 5% and 10%, respectively. # statistically insignificant.

Table 7. Augmented Excess Return Pricing Model with σ_{t-1}

Parameters	GARCH-M	GJR-GARCH-M	QGARCH-M	EGARCH-M
A. Daily				
c	0.554	0.519	0.112#	0.555
	(0.130)	(0.116)	(0.133)	(0.125)
d	-0.405	-0.378	0.029#	-0.432
	(0.127)	(0.113)	(0.137)	(0.123)
p	-0.135	-0.141	-0.141	-0.143
	[-0.158 -0.115]	[-0.164 -0.122]	[-0.162 -0.121]	[-0.164 -0.124]
$c^* = c + d$	0.149	0.141	0.141	0.123
$\xi = c + d + p$	0.014	0	0.000	-0.020
	[-0.028 0.053]	[-0.042 0.039]	[-0.042 0.041]	[-0.059 0.025]
corr(σ_t,σ_{t-1})	0.99	0.99	0.99	0.99
B. Weekly				
c	0.358**	0.177#	0.288#	0.249#
	(0.206)	(0.204)	(0.180)	(0.216)
d	0.040#	0.180#	0.072#	0.103*
	(0.194)	(0.192)	(0.172)	(0.045)
p	-0.365	-0.366	-0.365	-0.370
	[-0.417 -0.315]	[-0.415 -0.315]	[-0.420 -0.313]	[-0.420 -0.320]
$c^* = c + d$	0.398	0.357	0.360	0.352
$\xi = c + d + p$	0.033	-0.009	-0.005	-0.018
	[-0.068 0.129]	[-0.103 0.091]	[-0.109 0.098]	[-0.121 0.085]
corr(σ_t,σ_{t-1})	0.99	0.98	0.97	0.98
C. Monthly				
c	0.152#	0.154#	0.356#	0.175#
	(0.275)	(0.280)	(0.334)	(0.293)
d	0.359#	0.267#	0.070#	0.280#
	(0.279)	(0.293)	(0.436)	(0.318)
p	-0.456	-0.433	-0.430	-0.445
	[-0.581 -0.321]	[-0.542 -0.318]	[-0.522 -0.331]	[-0.559 -0.326]
$c^* = c + d$	0.511	0.421	0.426	0.455
$\xi = c + d + p$	0.055	-0.012	-0.004	0.010
	[-0.149 0.271]	[-0.214 0.181]	[-0.213 0.208]	[-0.223 0.234]
corr(σ_t,σ_{t-1})	0.96	0.95	0.95	0.95

Notes:

See Table 3

Regression is based on the augmented model $r_t = c\sigma_t + d\sigma_{t-1} + \alpha + br_{t-1} + u_t$

Computation of unconditional mean return $\mu = (\alpha + (c + d + p)\sigma)/(1-b)$

$c^* = c + d$ is for pure price of risk and p for skewness-kurtosis price of risk

$\xi = c + d + p$ is the combined pure-skewness-kurtosis price of risk

Table 8. Augmented Excess Return Pricing Model with σ_t^2

	GARCH-M	GJR-GARCH-M	QGARCH-M	EGARCH-M
A. Daily				
c	0.200	0.119*	0.078#	0.002#
	(0.058)	(0.048)	(0.059)	(0.041)
d	-0.028#	0.011#	0.035#	0.068
	(0.031)	(0.025)	(0.032)	(0.019)
p	-0.135	-0.143	-0.142	-0.146
	[-0.158 -0.115]	[-0.161 -0.121]	[-0.163 -0.123]	[-0.162 -0.125]
$\xi = c + d\sigma + p$	0.037	-0.013	-0.029	-0.077
	[-0.019 0.098]	[-0.057 0.029]	[-0.073 0.012]	[-0.122 -0.029]
corr(σ_t,σ_t^2)	0.91	0.90	0.91	0.88
B. Weekly				
c	0.352*	0.161#	0.117#	0.052#
	(0.173)	(0.190)	(0.189)	(0.161)
d	0.010#	0.040#	0.052#	0.065*
	(0.038)	(0.039)	(0.039)	(0.032)
p	-0.366	-0.370	-0.370	-0.378
	[-0.417 -0.314]	[-0.421 -0.320]	[-0.424 -0.315]	[-0.426 -0.324]
$\xi = c + d\sigma + p$	0.010	-0.115	-0.130	-0.172
	[-0.113 0.127]	[-0.229 0.008]	[-0.253 0.002]	[-0.261 -0.053]
corr(σ_t,σ_t^2)	0.94	0.94	0.94	0.93
C. Monthly				
c	0.412#	-0.132#	-0.171#	-0.192#
	(0.541)	(1.558)	(1.358)	(0.720)
d	0.007#	0.048#	0.055#	0.061#
	(0.055)	(0.145)	(0.125)	(0.068)
p	-0.445	-0.429	-0.435	-0.446
	[-0.571 -0.315]	[-0.538 -0.311]	[-0.526 -0.341]	[-0.564 -0.327]
$\xi = c + d\sigma + p$	0.003	-0.313	-0.322	-0.323
	[-0.152 0.148]	[-0.521 -0.058]	[-0.534 -0.041]	[-0.529 -0.044]
corr(σ_t,σ_t^2)	0.94	0.92	0.93	0.91

Notes:

See Table 3

Regression is based on the augmented model $r_t = c\sigma_t + d\sigma_t^2 + \alpha + br_{t-1} + u_t$

Computation of unconditional mean return $\mu = (\alpha + (c + p + d\sigma)\sigma)/(1-b)$

Figure 1. Skewness-Kurtosis Price of Risk

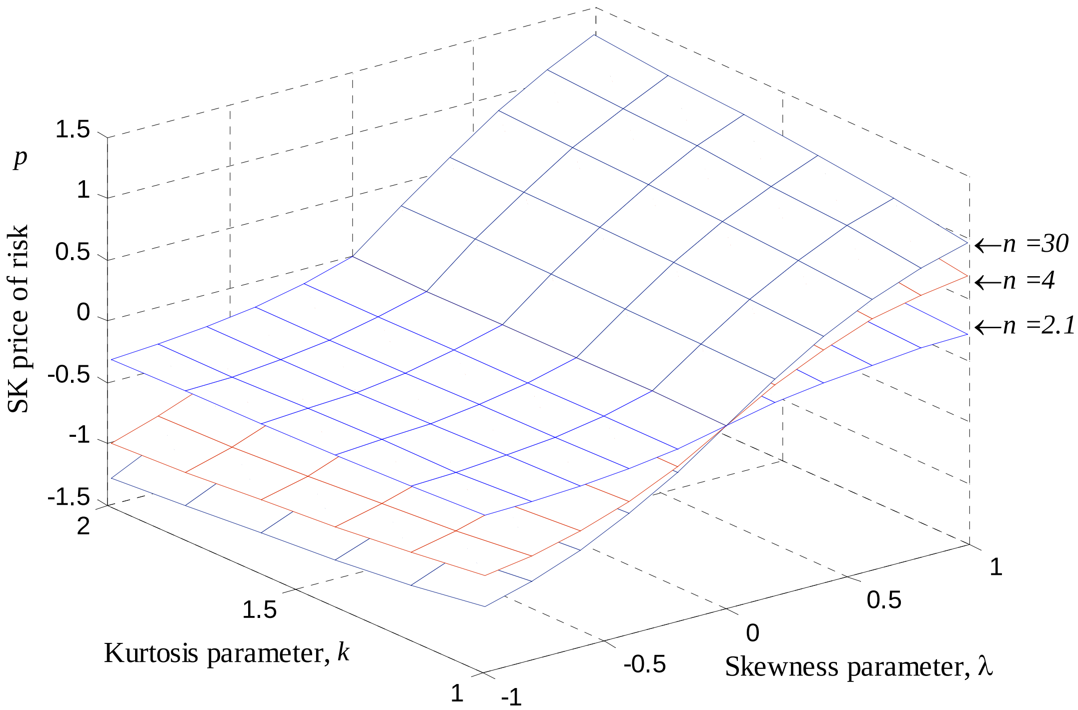

Figure 2. Pure and Skewness-Kurtosis Price of Risk: GJR Model

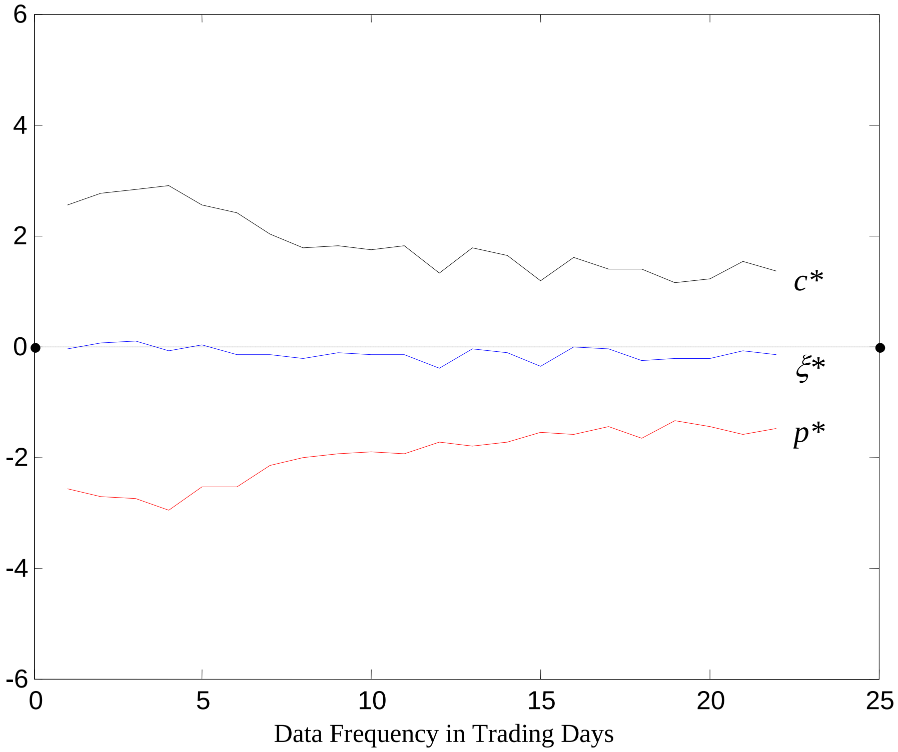

Figure 3. T-values for Pure and Skewness-Kurtosis Price of Risk: GJR Model